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Multipoint correlators of the impenetrable Bose gas

B T Matkarimov†

Institute of Nuclear Physics, Novosibirsk-90, Russia

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Abstract. The multipoint time-dependent correlation functions of the impenetrable Bose gas are considered. The determinant formula for the correlation functions with an arbitrary order of field operators is proposed.

1. Introduction

The one-dimensional non-relativistic Bose gas with point-like interaction between particles is a well-known model. It was intensively investigated by many authors for several decades [1–3, 8]. The Hamiltonian of this model is the following:

$$H=\int_{-\infty}^{+\infty}\mathrm{d}z(\Psi_z^+\Psi_z+c\Psi^+\Psi^+\Psi\Psi-h\Psi^+\Psi). \tag{1.1}$$

Here $\Psi^+(z)$, $\Psi(z)$ are canonical Bose fields, $[\Psi(x),\Psi^+(y)]=\delta(x-y)$, c is a coupling constant and h is a chemical potential. The model is generally considered in a box of length L with periodical boundary conditions. The impenetrable Bose gas corresponds to the model with an infinite value for the coupling constant, $c=+\infty$, and it is known to be equivalent to free fermions [1, 3, 8].

N-particle eigenfunctions of the Hamiltonian (1.1) at $c=\infty$ were constructed in [2]:

$$|\Psi_N\rangle=\frac{1}{\sqrt{N!}}\int\mathrm{d}z^N\chi_N(z_1,\dots,z_N|\mu_1,\dots,\mu_N)\Psi^+(z_1)\dots\Psi^+(z_N)|0\rangle. \tag{1.2}$$

Here

$$\chi_N(z|\mu)=\frac{1}{\sqrt{N!}}\prod_{N\geqslant i>k\geqslant 1}\varepsilon(z_i-z_k)\det[\exp(\mathrm{i}\mu_n z_m)]. \tag{1.3}$$

The norm of the wavefunction is given by $\langle\Psi|\Psi\rangle=L^N$. At zero temperature, $\mu_n=2\pi n/L$ by the periodic boundary conditions. The distribution of particles with momenta μ in the thermodynamic limit $N, L\to\infty$ with finite density $D=N/L$ in the state of the thermal equilibrium at temperature $T>0$ is given by the Fermi weight $\vartheta(\mu)=[1+\exp(E(\mu)/T)]^{-1}$, where the energy $E(\mu)=\mu^2-h$. At zero temperature, the momentum μ fills the interval $[-q,q]$ (Fermi sphere) with $q=\pi D$.

Recently, Its, Izergin, Korepin and Slavnov [4–6] represented the spacetime-dependent correlation functions of fields in an impenetrable Bose gas by the Fredholm

† Current address: Institute of High Energy Physics, Alma-Ata 480082, Kazakhstan.

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determinants. They also demonstrated that the potentials connected with correlators obey the system of nonlinear evolution equations. These results are of great interest in the theory of the one-dimensional Bose gas as well as in the general theory of the quantum and classical inverse spectral transform. Note that in [4-6], correlators of only a very special form were considered, namely the equal time temperature-dependent correlators of the form $\langle\Pi_{i=1}^{p}\Psi^{+}(x_i)\Pi_{j=p+1}^{2p}\Psi(x_j)\rangle$ and the general time-dependent correlators $\langle\Pi_{k=1}^{p}\Psi(x_{2k},t_{2k})\Psi^{+}(x_{2k-1},t_{2k-1})\rangle$.

In the present paper we will consider the multipoint time-dependent correlation functions with an arbitary order of fields

$$\langle\ldots\Psi_i\ldots\Psi_k^{+}\ldots\rangle=(\langle\Omega|\Omega\rangle)^{-1}\langle\Omega|\ldots\Psi_i(z_i,t_i)\ldots\Psi_k^{+}(z_k,t_k)\ldots|\Omega\rangle$$

where $|\Omega\rangle$ is the ground state with finite density and the number of Ψ or Ψ^{+} is equal to p by the conservation of particle numbers. There are C_{2p}^{p} correlation functions of such a type.

The method we use is closely related to that in [4-6]. But we use a different representation of the field form factor. This new representation allows us to find the determinant formulae for the correlators with an arbitrary order of fields. They are of the from

$$\langle\ldots\Psi(z_i,t_i)\ldots\Psi^{+}(z_k,t_k)\ldots\rangle=(2\pi)^{-p}(\mathrm{minor}\{W\})\det(I+K) \tag{1.4}$$

where minor (W) is the $p\times p$ minor of the $2p\times 2p$ matrix W (matrix of the potentials), and $\det(I+K)$ is the Fredholm determinant of the Fredholm operator $I+K$.

In the particular cases considered in [4-6] our formulae coincide with those in [4-6]. For the simplest correlators one has

$$\langle\Psi(0,0)\Psi^{+}(x,t)\rangle=(2\pi)^{-1}b_{++}(x,t)\det(I+K) \tag{1.5}$$

that is, the result of [5]. In the present paper we also find that

$$\langle\Psi^{+}(0,0)\Psi(x,t)\rangle=(2\pi)^{-1}b_{--}(x,t)\det(I+K). \tag{1.6}$$

The minors (potentials) b_{++} and b_{--} of the 2×2 matrix W obey the system of nonlinear differential equations that was also obtained in [4]; this system generalizes the fifth Painleve transcendent and is equivalent to the classical nonlinear Schrödinger equation. In the general case we demonstrate that the elements of matrix W obey the system of nonlinear multidimensional equations. The corresponding auxiliary linear problem is found.

In [7] the matrix Rieman problem for the multipoint equal time temperature correlators was constructed. We demonstrate that the matrix Rieman problem of such a type can be used for the time-dependent temperature correlators.

2. Multipoint correlation functions

Let us consider the multipoint time-dependent correlation functions with an arbitrary order of fields $\langle\ldots\Psi_i\ldots\Psi_k^{+}\ldots\rangle=(\langle\Omega|\Omega\rangle)^{-1}\langle\Omega|\ldots\Psi_i(z_i,t_i)\ldots\Psi_k^{+}(z_k,t_k)\ldots|\Omega\rangle$. The spacetime dependence of the field operators in the correlators is ordered in the following manner: the first left field operator depends on x_1, t_1, the second depends on x_2, t_2, and so on.

The function $X_N(x,t,\lambda)$ with arbitrary $\lambda_n=2\pi n/L$ forms the complete set of functions. The first step to calculate the correlator is to insert the unit operators $I=(\langle\Psi_N|\Psi_N\rangle)^{-1}\Sigma_\lambda|\Psi_N\rangle\langle\Psi_N|$ into the finite-particle correlator $\langle\ldots\rangle_N=\langle\Psi_N|\ldots|\Psi_N\rangle/\langle\Psi_N|\Psi_N\rangle$ and then calculate the thermodynamic limit L, $N\to\infty$. The form factor $F(x,t|\lambda,\mu)=\langle\Psi_{N+1}(\lambda)|\Psi^+(x,t)|\Psi_N(\mu)\rangle/\langle\Psi_N(\mu)|\Psi_N(\mu)\rangle$ was calculated in [5]. We represent it in a different form by introducing the fictitious momentum μ_{N+1}. This trick is very useful for the following calculations:

$$F(x,t|\lambda,\mu)=\left(\frac{2\mathrm{i}}{L}\right)^N\left(\prod_{i=1}^{N+1}e(x,t,\lambda_i)\prod_{j=1}^{N}e^*(x,t,\mu_j)\right)$$
$$\times\operatorname*{Lim}_{\mu_{N+1}\to\infty}\left((-\mu_{N+1})\det{}^{N+1}\left(\frac{1}{\lambda_n-\mu_m}\right)\right)\tag{2.1}$$

where

$$e(x,t,\lambda)=\exp[\mathrm{i}(t\lambda^2-x\lambda)]\tag{2.2}$$

$e^*(x,t,\lambda)$ is the complex conjugate of $e(x,t,\lambda)$ and $\det^N\{a_{nm}\}$ is the determinant of the $N\times N$ matrix A with elements a_{nm}.

So, the correlator $\langle\ldots\rangle_N$ can be represented as a sum over the form factor product, i.e.

$$\left\langle\prod_{i=1}^{p}\Psi_i^+\prod_{j=p+1}^{2p}\Psi_j\right\rangle_N=\frac{1}{L^p}\sum\left(\prod_{i=1}^{p}F(x_i,t_i|\lambda_{i-1},\lambda_i)\prod_{j=p+1}^{2p}F^*(x_j,t_j|\lambda_{j-1},\lambda_j)\right)\tag{2.3}$$

where $\Psi_i=\Psi(x_i,t_i)$, $\Psi_i^+=\Psi^+(x_i,t_i)$ and the summation is over $\lambda_n^i=2\pi n/L$ where $i=1,\ldots,2p-1$, n is an integer, and the sets $\{\lambda^0\}$ and $\{\lambda^{2p}\}$ are equivalent. The sets $\{\lambda^0\}$ and $\{\lambda^{2p}\}$ directly correspond to the ground state $|\Omega\rangle$, and at zero temperature these sets are finite: $\lambda_{\max}\sim q$, $\lambda_{\min}\sim -q$.

To calculate (2.3) one has to represent the determinant product in (2.3) as the determination of one matrix. Since the matrices in (2.3) are of different ranks it is necessary to decompose some determinants over one or more columns or rows. As was done in [5], we use the following relation:

$$\sum_\lambda\left[\left(\prod_{i=1}^{N}e(\lambda_i)\right)\det{}^N\left\{\frac{1}{\mu_n-\lambda_k}\right\}\det{}^N\left\{\frac{1}{\lambda_n-\mu_n}\right\}\right]=\det{}^N\left\{\sum_{\lambda_k}\frac{e(\lambda_k)}{(\mu_n-\lambda_k)(\lambda_k-\mu_k)}\right\}.\tag{2.4}$$

Using the technique of [5] we calculate the sum in (2.3) over a λ^n where n is an odd number. There are four different cases which correspond to the following combinations of the field operators: $\Psi^+\Psi^+$, $\Psi^+\Psi$, $\Psi\Psi^+$, $\Psi\Psi$. The direct calculation demonstrates that after summation on λ^n and calculation of an infinite limit for all fictitious momenta, the result can be presented as the convolution of some universal matrix Δ_{nm} with four different vector functions.

Let us introduce the following notation and the functions $E_{ik}(\lambda)$ and $G_{ik}(\lambda)$:

$$e_i(x_i,t_i|\lambda)=e(x_i,t_i,\lambda)\tag{2.5}$$

$$e_{ik}(x_i,t_i,x_k,t_k|\lambda)=e_i(\lambda)e_k^*(\lambda)\tag{2.6}$$

$$E_{ik}(x,t|\lambda)=\sum_\mu\frac{2}{L}\frac{e_{ik}(\mu)}{\mu-\lambda}\tag{2.7}$$

$$G_{ik}(x,t)=\sum_\mu\frac{2}{L}e_{ik}(\mu)\tag{2.8}$$

where $\mu_n = 2\pi n/L$ and $n \in \mathbb{Z}$. Then we have

$$\sum_{\lambda} F(x_s, t_s|\mu, \lambda) F(x_p, t_p|\lambda, \nu) = \sum_{n,k} e_s(\mu_n) e_s(\mu_k) E_{ps}(\mu_k) \Delta^{n,k}$$

$$\sum_{\lambda} F(x_s, t_s|\mu, \lambda) F^*(x_p, t_p|\lambda, \nu) = \sum_{n,k} e_s(\mu_n) e_p^*(\nu_k) \Delta_k^n$$

$$\sum_{\lambda} F^*(x_s, t_s|\mu, \lambda) F^*(x_p, t_p)|\lambda, \nu) = -\sum_{n,k} e_p^*(\nu_n) E_{ps}(\nu_n) e_p^*(\nu_k) \Delta_{n,k} \tag{2.9}$$

$$\sum_{\lambda} F^*(x_s, t_s|\mu, \lambda) F(x_p, t_p|\lambda, \nu) = \frac{L}{2} G_{ps}\Delta - \sum_{n,k} E_{ps}(\mu_n) e_s(\mu_n) E_{ps}(\nu_k) e_p^*(\nu_k) \Delta_k^n$$

where Δ_m^n is a minor of the matrix K_{nm}, which does not have an nth row and mth column:

$$K_{nm} = e_{ps}(\mu_n)\delta_{\mu_n,\nu_m} + \frac{2}{L}\frac{E_{ps}(\mu_n) - E_{ps}(\nu_m)}{\mu_n - \nu_m}. \tag{2.10}$$

Here $\delta_{\mu,\nu}$ is the Kroneker symbol.

Summation over other λ_i (i is an even number) is carried out in the same manner. We have the correlator as a convolution of the $p \times p$ minor of some matrix K_{mn} with some functions $e_n^+(\lambda)$, $e_m^-(\lambda)$. In the next step one can decompose the $p \times p$ minor of the matrix K over 1×1 minors by the Leibnitz rule and calculate the thermodynamic limit. In this limit [5]

$$\sum_{\lambda} \frac{2\pi}{L}(\ldots) \to \int d\lambda(\ldots). \tag{2.11}$$

Note that the final summations are made on the finite interval $[-q, +q]$, and in the thermodynamic limit we represent the 1×1 minor as the resolvent of the Fredholm operator.

Let us introduce the following functions:

$$E_{ps}(x_p, x_s, t_p, t_s|\lambda) = \int_{-\infty}^{+\infty} d\mu \frac{e_{ps}(\mu)}{\mu - \lambda} \tag{2.12}$$

$$G_{ps}(x_p, x_s, t_p, t_s) = \int_{-\infty}^{+\infty} d\mu\, e_{ps}(\mu) \tag{2.13}$$

Let us also introduce the linear integral operator with a kernel:

$$K(\lambda_0, \lambda_p) = \int_{-\infty}^{+\infty}\int d\lambda_1 \ldots d\lambda_{p-1}$$
$$\times\left(\prod_{m=1}^{p}(\delta(\lambda_m - \lambda_{m-1}) + K_m(\lambda_m, \lambda_{m-1}) e_{2m}^*(\lambda_m) e_{2m-1}(\lambda_{m-1}))\right) \tag{2.14}$$

where

$$K_m(\lambda, \mu) = \frac{E_{2m,2m-1}(\lambda) - E_{2m,2m-1}(\mu)}{\pi^2(\lambda - \mu)}. \tag{2.15}$$

The kernel $K(\lambda, \mu)$ can be represented by the following:

$$K(\lambda, \mu) = \delta(\lambda - \mu) + \frac{1}{\lambda - \mu}\sum_{s=1}^{2p}(-1)^s e_s^+(\lambda) e_s^-(\mu) \tag{2.16}$$

where the functions $e_s^+(\lambda)$, $e_s^-(\lambda)$ are defined as

$$
\begin{aligned}
e_{2s-1}^+(\lambda_0) &= \frac{1}{\pi}\int \prod_{k=1}^{s-1} \mathrm{d}\lambda_k \left(\prod_{m=1}^{s-1} S_m(\lambda_m, \lambda_{m-1})\right) e_{2s-1}(\lambda_{s-1}) \\
e_{2s-1}^-(\lambda_p) &= \frac{1}{\pi}\int \prod_{k=s}^{p-1} \mathrm{d}\lambda_k \left(\prod_{m=s+1}^{p} S_m\right) E_{2s,2s-1}(\lambda_s) e_{2s}^*(\lambda_s) \\
e_{2s}^+(\lambda_0) &= \frac{1}{\pi}\int \prod_{k=1}^{s-1} \mathrm{d}\lambda_k \left(\prod_{m=1}^{s-1} S_m\right) E_{2s,2s-1}(\lambda_{s-1}) e_{2s-1}(\lambda_{s-1}) \\
e_{2s}^-(\lambda_p) &= \frac{1}{\pi}\int \prod_{k=s}^{p-1} \mathrm{d}\lambda_k \left(\prod_{m=s+1}^{p} S_m(\lambda_m, \lambda_{m-1})\right) e_{2s}^*(\lambda_s).
\end{aligned}
\tag{2.17}
$$

Here $s = 1, \ldots, p$ and

$$S_m(\lambda_m, \lambda_{m-1}) = \delta(\lambda_m - \lambda_{m-1}) + K_m(\lambda_m, \lambda_{m-1}) e_{2m}^*(\lambda_m) e_{2m-1}(\lambda_{m-1}).$$

The Fredholm operator (2.14), acting on the finite interval $[-q, +q]$, was introduced in [4], its kernel $K(\lambda, \mu)$ (2.14) has a very special form—it is the convolution of similar kernels. This property can be used to prove (2.16) by induction. We also introduce the functions $f_s^+(\lambda)$, $f_s^-(\lambda)$ as solutions of the following integral equations:

$$f_s^+(\lambda) + \sum_{p=1}^{2p} (-1)^p e_p^+(\lambda) \int_{-q}^{+q} \frac{\mathrm{d}\mu}{\lambda - \mu} e_p^-(\mu) f_s^+(\mu) = e_s^+(\lambda) \tag{2.18}$$

$$f_s^-(\lambda) + \sum_{p=1}^{2p} (-1)^p e_p^+(\lambda) \int_{-q}^{+q} \frac{\mathrm{d}\mu}{\lambda - \mu} e_p^-(\mu) f_s^-(\mu) = e_s^-(\lambda). \tag{2.19}$$

The integral operator with the kernel $R(\lambda, \mu)$,

$$R(\lambda, \mu) = \delta(\lambda - \mu) - \frac{1}{\lambda - \mu} \sum_{s=1}^{2p} (-1)^s f_s^+(\lambda) f_s^-(\mu) \tag{2.20}$$

is the resolvent of the operator $K(\lambda, \mu)$. We then introduce the $2p \times 2p$ matrices V and H:

$$V_{nm}(x, t) = (-1)^n \int_{-q}^{+q} \mathrm{d}\mu\, e_n^-(\mu) f_m^+(\mu) \tag{2.21}$$

$$H_{nm}(x, t) = \begin{cases} G_{nm}(x, t) & \text{where } n = 2k-1,\ m = 2k,\ k \in \mathbb{N} \\ 0 & \text{other } n, k. \end{cases} \tag{2.22}$$

The $2p$ correlation functions correspond to the $p \times p$ minors of the matrix $V + H$.

The formulae (2.9) allow us to establish the following role to obtain a minor of the potential's matrix $V + H$ which corresponds to a given correlation function: if the ith field operator in correlator is $\psi(\psi^+)$ we delete the ith column (row) in the matrix $V + H$. Remained rows and columns form the mentioned minor, i.e. $\langle\psi\psi^+\rangle \sim (V+H)_{12}$, $\langle\psi^+\psi\rangle \sim (V+H)_{21}$.

$$\langle \ldots \rangle = (2\pi)^{-p} (\mathrm{minor}(V + H)) \det K(\lambda, \mu). \tag{2.23}$$

The temperature correlation functions are similar to the obtained correlators. In the case of non-zero temperatures, the Fredholm operator with the kernel $K(\lambda, \mu)$ acts on the whole axis, but the integral part of the kernel $K(\lambda, \mu)$ must be changed [5, 8]:

$$K(\lambda, \mu) = \delta(\lambda - \mu) + \frac{1}{\lambda - \mu} \sum_{s=1}^{2p} (-1)^s e_s^+(\lambda) \vartheta^{1/2}(\lambda) e_s^-(\mu) \vartheta^{1/2}(\mu) \tag{2.24}$$

where $\vartheta(\lambda)$ is the Fermi weight.

The functions $f^+(x, t|\lambda), f^-(x, t|\mu)$ form the kernel of the resolvent of the operator $K(\lambda, \mu)$, similarly to (2.18) and obey the following equations:

$$f_s^+(\lambda) + \sum_{p=1}^{2p} (-1)^s e_p^+(\lambda) \int_{-\infty}^{+\infty} \frac{\mathrm{d}\mu}{\lambda - \mu} \vartheta(\mu) e_p^-(\mu) f_s^+(\mu) = e_s^+(\lambda) \tag{2.25}$$

$$f_s^-(\lambda) + \sum_{p=1}^{2p} (-1)^s e_p^+(\lambda) \int_{-\infty}^{+\infty} \frac{\mathrm{d}\mu}{\lambda - \mu} \vartheta(\mu) e_p^-(\mu) f_s^-(\mu) = e_s^-(\lambda). \tag{2.26}$$

The matrix of the potentials V_{nm} is now defined as the following:

$$V_{nm}(x, t) = (-1)^n \int_{-\infty}^{+\infty} \mathrm{d}\mu \, \vartheta(\mu) e_n^-(x, t|\mu) f_m^+(x, t|\mu). \tag{2.27}$$

Then (2.23) with rescaled $K(\lambda, \mu)$, $V_{nm}(x, t)$ is valid for the temperature correlation functions.

3. Differential equations for the correlation functions

The Fredholm determinant representation for the correlation functions permits us to derive nonlinear differential equations for the potentials V_{nm} [4]. Let us consider (2.18) as a system of integral equations for the vector function $F(x, t|\lambda) = (f^+(\lambda_1), \ldots, f^+(\lambda_{2p}))$. Differentiating this system with respect to x_n and t_m and using the Fredholm alternative we derive the system of equations

$$L_n(d_{xn}, V_{ik}) F(x, t|\lambda) = 0 \tag{3.1}$$

$$M_m(d_{tm}, V_{ik}) F(x, t|\lambda) = 0 \tag{3.2}$$

where L_n and M_m are linear differential matrix operators, $n, m = 1, \ldots, 2p$, $d_{xn} = \mathrm{i}\,\mathrm{d}/\mathrm{d}x_n$ and $d_{tn} = \mathrm{i}\,\mathrm{d}/\mathrm{d}t_n$.

The compatibility conditions of systems (3.1) and (3.2) lead us to the multi-dimensional nonlinear differential equations for the potentials V_{nm}.

To derive the matrix operators L and M one can use the following relations:

$$d_{xn} e_k^+(x, t|\lambda) = \lambda \delta_{nk} e_k^+(\lambda) + A_{ks}^n(x, t) e_s^+(\lambda) \tag{3.3}$$

$$d_{xn} K(\lambda, \mu) = (-1)^n e_n^+(\lambda) e_n^-(\mu) \tag{3.4}$$

$$d_{tn} e_k^+(x, t|\lambda) = -\lambda^2 \delta_{nk} e_k^+(\lambda) - \lambda A_{ks}^n e_s(\lambda) + B_{ks}^n(x, t) e_s^+(\lambda) \tag{3.5}$$

$$d_{tn} K(\lambda, \mu) = -(-1)^n (\lambda + \mu) e_n^+(\lambda) e_n^-(\mu) - (-1)^p A_{ps}^n e_s^+(\lambda) e_p^-(\mu). \tag{3.6}$$

Here $A^n(x, t)$ and $B^n(x, t)$ are some matrices which are defined by (3.3)-(3.6), and we assume summation over the indices s and p.

So, (3.1) and (3.2) have the following form:

$$d_{xn} f_k^+(\lambda) = \lambda \delta_{nk} f_k(\lambda) + (\delta_{nk} V_{sk} - \delta_{ns} V_{nk} + A_{ks}^n) f_s^+(\lambda) \tag{3.7}$$

$$d_{tn} f_k(\lambda) = -\lambda^2 \delta_{nk} f_k(\lambda) - \lambda(\delta_{nk} V_{sk} - \delta_{ns} V_{nk} + A_{ks}^n) f_s^+(\lambda)$$

$$+(V_{nk}V_{sn}-\delta_{nk}V_{pk}V_{sp}-\delta_{nk}W_{sk}+\delta_{ns}W_{nk}$$
$$+A^n_{ps}V_{pk}-A^n_{kp}V_{sp}+B^n_{ks})f_s(\lambda) \tag{3.8}$$

where we assume summation over indices s and p, and

$$W_{nk}=(-1)^n\int \mathrm{d}\mu\ \mu e^-_n(\mu)f^+_k(\mu). \tag{3.9}$$

The temperature correlation functions can be described in terms of the matrix Riemann problem, similarly to the equal time correlators [7]: one has to find the $2p\times 2p$ matrix valued function $X(\lambda)$ which is holomorphic for $\mathrm{Im}\ \lambda<0$ and $\mathrm{Im}\ \lambda>0$, and are related at the real axes by

$$X^-(\lambda)=X^+(\lambda)G(\lambda) \tag{3.10}$$

where $X^\pm(\lambda)=\lim X(\lambda\pm\mathrm{i}\varepsilon)$, $\varepsilon\to 0$, $\varepsilon>0$, and $X(\lambda)$ is equal to the unit matrix at $\lambda\to\infty$: $X(\infty)=I$. The conjugating matrix $G(\lambda)$ has the following elements:

$$G_{nk}(x,t|\lambda)=\delta_{nk}+(-1)^n e^+_n(x,t|\lambda)e^-_k(x,t|\lambda)\vartheta(\lambda). \tag{3.11}$$

The matrix Riemann problem is equivalent to the system of singular integral equations:

$$X^+(\lambda)=I+\frac{1}{2\pi\mathrm{i}}\int \mathrm{d}\mu\frac{X^+(\mu)(I-G(\mu))}{\mu-\lambda-\mathrm{i}0}. \tag{3.12}$$

Equations (3.12) with the conjugating matrix G (3.11) are equivalent to the system of equations (2.25) [7]. The matrix $X(\lambda)$ at $\lambda\to\infty$ can be expanded as

$$X(\lambda)=I+\Psi 1(x,t)/\lambda+\ldots. \tag{3.13}$$

The potentials V_{nk} are related to the matrix $\Psi 1(x,t)$ [7]:

$$V_{nk}=-\mathrm{i}\Psi 1_{nk}(x,t). \tag{3.14}$$

Note that the adjoint Fredholm operator $K^{\mathrm{T}}(\lambda,\mu)=K(\mu,\lambda)$ (see (2.14)) also describes quantum $2p$ point correlation functions but with a different spacetime dependence, $x_i\to -x_{2p+1-i}$, $t_i\to -t_{2p+1-i}$. The general index of the matrix Riemann problem (3.10) is zero.

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